(#8-1) There is a circle with 6 numbers placed along it. In the beginning, these numbers are:

```
  2  4
0      6
 10  8
```

In this solitaire game, you make a move by adding "1" to each of any 2 adjacent numbers. You win the game if you make all 6 numbers equal to 2004.

Can this game be won? Either describe the winning strategy, or prove that it is impossible to win.

(#8-2) Believe it or not, the expression $\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}$ is constant for x in some interval, a < x < b. Find a, b, the constant value, and explain why this is true.

(#8-3) A man finishes work early, and so he takes the early train home, arriving at the train station one hour earlier than normal. He knows that his wife is planning to pick him up at the normal time, so he begins to walk home. His wife sees him on the road, and picks him up. They arrive home 20 minutes earlier than normal. How long was the man walking?

(#8-4) The natural numbers m and n are relatively prime. Prove that the greatest common divisor of (m+n) and (m^2+n^2) is either 1 or 2.

(#8-5) A pool table is a square, each side is 1 yard long. It has 4 holes: 1 in each corner. Going clockwise, the corners are named A, B, C, and D. A ball is positioned at corner A. You need to shoot the ball in such a way that it would bounce off point E on side BC, than bounce off point F on side CD, than bounce off point G on side DA, than fall into hole at corner B. Where should you aim? (Which means - you need to find exactly where the point E should be).

(#8-6) In this game for 2 players, one starts by naming a positive integer A, and then they alternate making moves. A move consists of naming a new positive number B which is smaller than the current number A, and is such that A is divisible by (A-B). The player who names the number "1" is the winner.
For every starting number determine which of the 2 players has the winning strategy.

(#8-7) On a plane, there is 1 straight line (representing a brook) plus 2 points, both on the same side of the brook, representing a house and a barn. Every day a girl takes a bucket, walks from the house to the brook to get water, and then brings it to the horse who is in the barn. Draw the shortest path for her to walk (you may use a compass and a ruler).

(#8-8) A rectangular chocolate bar is divided by grooves into 2004 x 2005 little squares. Your task is to completely break it, along the grooves only, into 4,018,020 individual little square pieces. While you are doing it, at any given point, there would be several rectangular pieces on the table, and you are allowed to pick up any one piece only, then make only one complete straight cut along any groove you choose (to break that piece into 2 smaller pieces), then you put them back on the table. This counts as 1 move. What's the smallest number of moves you need to complete your task?

(#8-9) A dentist has 3 patients waiting, but he has only 2 pairs of gloves left. He wants to treat all 3 patients, and he must use both of his hands while treating each patient. Is there a way to do it?

(#8-10) You have a 4x4 square table with 6 Xs drawn in it (no more than 1 X per square). Prove that you can always find 2 columns and 2 rows, such, that if you erase them, then there will be no Xs left.
Find how 7 Xs can be drawn in such a way, that no matter which 2 columns and 2 rows you choose and erase, there will always remain at least 1 X.

(#8-11) Imagine that you are color-blind (can't tell green from red). There are 2005 identical red/green game pieces from the game "Reversi" (a.k.a. "Othello") on a table in front of you. These look the same as pieces used in Checkers, except one side of each is green, and the other side is red.

You know that 1003 pieces are red-on-top, and the remaining 1002 pieces are green-on-top. You are welcome to play with all the pieces for as long as you want. You may flip them, write on them, discard some, etc. Your task is to select 2 groups of pieces A and B (A and B together may contain any number of pieces - from 1 up to 2005) so that the number of green-on-top pieces in A would be equal to the number of green-on-top pieces in B. Can you do it?

(#8-12) A one-storey apartment building is an 2004x2004 square. Nobody likes his own apartment, and every tenant wants to move into any one of his four immediate neighbor's apartments. Is it possible to move all 4,016,016 tenants so everyone will be happy (each one will move into one of his four neighbors' apartments)? Same question for a 2005x2005 square.

(#8-13) Does there exist a function $\mathbf{f}: \mathbf{Q} \rightarrow \mathbf{Q}$ (where $\mathbf{Q}$ is the set of all rational numbers), such that for any rational number x, $\mathbf{f}(\mathbf{f}(x)) = -x$?

(#8-14) A fuse (as in "a combustible cord for setting off an explosive") is called "regular" if its total burning time is exactly 1 hour. However, it may burn unevenly (faster in some places and slower in others). You have 2 regular fuses, matches, and a ruler (but no clocks). Can you measure a time interval of precisely 45 minutes?

(#8-15) At the center of an Island, which is shaped as a circle, there is a Deer. There is a Fence all around the Island, which is surrounded by Water. Deer can jump over the Fence and swim away, but there is a Wolf just outside the Fence. Wolf can neither jump, nor swim. Deer and Wolf can always see each other. If Deer jumps over the Fence at a point where Wolf is standing, then Wolf will eat him. Wolf can run exactly 4 times as fast as Deer. Can Deer escape? (Note: length of any circle divided by length of its diameter equals approximately 3.1416).

(#8-16) Say that we have a hallway with 2004 lockers, numbered sequentially from 1 to 2004. The lockers have two possible states, open and closed. Initially all the lockers are closed. The first kid who walks down the hallway flips every locker to the opposite state, that is, opens them all. The second kid flips the first locker door and every other locker door to the opposite state, that is, closes them. The third kid flips the first locker door and every third door, opening some, closing others. The forth kid does every fourth door, etc. After 2004 kids have passed down the hallway, which lockers are open, and which are closed?

(#8-17) You have a (2004 x 2005) square table of integers. To make 1 move, you select either 1 row, or 1 column, and then change the sign of all numbers in it. Is it true that it is always possible to make a few moves so that as a result the sum of numbers in each row and the sum of numbers in each column (that's a total of 4009 sums) will all be non-negative?

(#8-18) In the Martian language there are only two letters, **a** and **b**, and it is postulated that the letter **a** is a word. Furthermore, all additional words are formed according to the following rules:

- **A** Given any word, a new word can be formed from it by adding a **b** at the right hand end.
- **B** If in any word a sequence **aaa** appears, a new word can be formed by replacing **aaa** by the letter **b**.
- **C** If in any word a sequence **bbb** appears, a new word can be formed by omitting **bbb**.
- **D** Given any word, a new word can be formed by writing down the sequence that constitutes the given word twice.

For example, by (D), **aa** is a word, and by (D) again, **aaaa** is a word. Hence by (B) **ba** is a word, and by (A), **bab** is also a word. Again, by (A), **babb** is a word, and so by (D), **babbbabb** is also a word. Finally, by (C) we find that **baabb** is a word.
Prove that in this language **baabaabaa** is not a word.

(#8-19) In a group of students, 50 speak English, 50 speak French and 50 speak Spanish. (Some students speak more than one language). Prove that it is possible to divide the students into 5 groups (not necessarily equal), so that in each group 10 speak English, 10 speak French and 10 speak Spanish.

(#8-20) A giant kangaroo is attached to a pole by a rubber band, and there is a flea sitting on that pole. The kangaroo jumps 1 mile, stretching the rubber band. Then the flea jumps 1 inch towards the kangaroo and lands on the rubber band. Then the kangaroo (who is still attached by the rubber band to the pole) again jumps 1 mile (continuing in the same direction). Then the flea again jumps 1 inch towards the kangaroo and again lands on the rubber band. So they continue, in a straight line, taking turns. The rubber band can stretch forever. The Earth is a flat and infinite plane. Will the flea ever catch up with the kangaroo?

(#8-21) There are 2004 gas cans placed in various spots along the side of a circular one-way road. Each can has some gas in it, and the total amount of gas is sufficient to drive a car one full circle along that road. Prove that there exists at least 1 lucky gas can, such that if there is a car with an empty gas tank parked right next to that lucky can, then you would be able to drive the full circle (while picking up every gas can you find along your way).

(#8-22) At a bank, there is a safe with several locks on it, and there are n employees. Each employee has keys to some of the locks. You want any k of the employees (where $1 \le k \le n$), when they are together, to be able to open the safe, but no (k-1) employees should be able to do it. What's the minimum number of locks for which this can be done?

(#8-23) On a plane, there are 2 parallel straight lines (representing a brook) plus 2 points, on the opposite sides of the brook representing a house and a barn. Every day a girl takes a bucket, walks from the house to the brook to get water, and then brings it to the horse who is in the barn. When she reaches the brook, she always crosses it by the shortest possible route (which is a segment perpendicular to the brook's shores). Draw the shortest path for her to walk (you may use a compass and a ruler).

(#8-24) An 8x8 chessboard is empty except for 2 pawns which occupy 2 diagonally opposite corners (A1 and H8). You also have 31 dominos of the size 2x1. Is it possible to completely cover the rest of the chessboard with these dominos by placing each on some 2 adjacent squares?

(#8-25) You have a sheet of graph paper, a sheet of transparent paper, and a little bit of red paint. The amount of your paint is such that it's not enough to completely fill in a 1x1 square on your graph paper. Now, all your paint got spilled onto your sheet of transparent paper. Prove that it is possible to place it on top of the graph paper in such a way that the paint won't cover any of the graph paper's vertices (points where graph lines intersect).

(#8-26) In Afganistan there are 6 cities and 2 airlines. Every pair of cities is served by at least 1 of the 2 airlines with a non-stop flight. Prove that it is possible to select 3 cities so that every pair of them (that's 3 pairs) is served by the same airline.

(#8-27) There are 2004 trees arranged in a straight line, and on each of them there sits 1 bird. Every minute 2 of the birds take off and each of them lands on a tree next to the one where it sat before. However, the 2 birds always fly in the opposite directions (one to the right, the other one to the left). Can all 2004 birds eventually gather together on the same tree?

(#8-28) You have a 2005 x 2005 square table. There is either 1 or -1 in its every square. A product of the numbers in each column is written under that column. A product of the numbers in each row is written to the right of that row. Is it possible that the sum of these 4010 products equals 0 ?

(#8-29) In this game for 2 players, there are 2004 stones arranged in a circle. To make a move, one must remove and discard either 1, or 2 stones. But, if one removes 2 stones, they must be adjacent to each other, with no gap between them ("adjacent" means they were adjacent since the beginning of the game). The player who removes the last stone, wins. Which player has a winning strategy? Same question if you start with 2005 stones.

(#8-30) To ensure that all students are visible in a photo, the photographer arranged a group of 2004 students into two equal rows so that every student in the back row is taller than the student standing in front of her in the front row. As he was about to shoot the photo, the principal walked in and ordered each row rearranged so that the students in it increase in height from left to right, while everyone remains within her original row. Prove that photographer can still take his picture now (all students are still visible).

(#8-31) A class of 33 students is having a meeting to decide who to vote for. In the beginning, 12 students were going to vote for Bush, 11 for Kerry, and 10 for Nader. The meeting consists of a series of one-on-one conversations. Every time any pair of students have a conversation, the following happens: if their current votes are the same, then they don't change them. If they are different (say, Bush and Kerry), then they both switch to the third vote (Nader). Can they eventually achieve a unanimous vote?

(#8-32) Take a map (a plane), and from each town (a point) shown on it draw an arrow pointing to its nearest neighbor. Assume there are 2005 towns on this map, and each different pair of towns is a different distance apart (note there are a total of 2005 arrows, any pair of towns may be connected by 0, 1, or 2 arrows). Prove that there exists at least 1 town that has no incoming arrows.

(#8-33) A tobacco company's CEO submitted an affidavit to the court consisting of 2004 statements; it begins like this:

There is 1 false statement in this affidavit. There are 2 false statements in this affidavit. There are 3 false statements in this affidavit. There are 4 false statements in this affidavit… … …

………… and so it goes on and on, and it ends like this:

There are 2002 false statements in this affidavit. There are 2003 false statements in this affidavit. There are 2004 false statements in this affidavit.

What is the total number of false statements in the affidavit?

(#8-34) Is it possible to put some pawns on an 8x8 board so that there would be exactly 5 pawns in every 3x3 square, and there would be exactly 4 pawns in every 2x4 rectangle (both vertical and horizontal ones).

(#8-35) Prove that the sum **1/2 + 1/3 + 1/4 + … + 1/n** is never an integer for any positive integer **n**.

(#8-36) In a group of 2004 students each has at least 1002 friends (friendship is always mutual). Prove that all 2004 can be seated at a round table so that every pair of neighbors are friends.

Give an example of such group of 2004 students in which each has at least 1001 friends, but no matter how you seat them at a round table, there will always be at least 2 neighbors who are enemies.

(#8-37) An "8x8" chessboard is empty except for a single pawn. You also have 21 "3x1" rectangles called “triminos”. Find all squares on the chessboard such that when that single pawn is placed on it, then the rest of the board can be completely covered by placing each "3x1" trimino on some 3 adjacent squares.

(#8-38) 2004 students are standing in a single line, all facing the Principal. The Principal said: “Everybody turn to the left” after which some students turned to the left, some turned to the right, and some didn’t move. Is it always true that the Principal can now insert herself into the line in such a way that the numbers of students facing her on both of her sides will be the same?

(#8-39) A giant kangaroo and a flea are sitting on a straight road that goes all the way around the Earth’s equator (and returns to the same point). It’s exactly 40,000 kilometers long. The giant kangaroo jumps exactly $\sqrt{2}$ kilometers along the road. Then the flea jumps 1 meter in the same direction. Then kangaroo again jumps $\sqrt{2}$ kilometers (continuing in the same direction), then the flea jumps 1 meter, again in the same direction. So they continue, along the endless road, taking turns. Prove that sooner or later the kangaroo is going to step on the flea.

(#8-40) There is a set of dominos glued to a flat surface; they do not overlap. 2 dominos are called “adjacent” if they touch each other along parts of their borders (at more than 1 point). Let’s call a coloring of a set of dominos “regular” (with each domino painted with a single color) if any 2 adjacent dominos are painted in different colors. Give an example of such a set that any regular coloring of it requires at least 4 colors.

(#8-41) In a country there are 2005 cities and 1 airline which has some daily non-stop flights connecting some pairs of cities. For any 2 given cities, there exists precisely one itinerary (a sequence of flights) which connects them while not going through any city more than once. (Of course, a passenger may have to wait overnight for connections at some cities). Prove that the total number of daily non-stop flights in this country equals 2004.

(#8-42) On a sheet of paper, every point is painted in 1 of 3 colors. Prove that there exist 2 points of same color such that the distance between them equals 1 inch.

(#8-43) There are 4 people in a family. If the daughter's salary is doubled, then total family income will increase by 5%. If, instead, Mom's salary is doubled, it would increase by 15%. If Dad's salary is doubled, it would increase by 25%. By what percentage would the total income increase if Grandpa's pension is doubled?

(#8-44) Give an example of 5 numbers the product of which does not equal 0, and such that if you subtract 1 from each number, then their product will remain the same.

(#8-45) Do there exist 2 sets A and B on the plane such that B is a subset of A, but A does not equal B; A is bounded (lies entirely inside some circle); and A and B are congruent?

(#8-46) N students sit around a round table. Each has a number starting with #1, and then, going clockwise, the numbers are: #2, #3, ..., #N. The teacher skips student #1, and asks student #2 to leave. He then proceeds clockwise, always skipping one student, and asking the next one to leave (#4, #6, etc.). After making a full circle, the teacher continues clockwise around the table until there is only one student left, and that student is the winner. For each N find F(N) - the number of the student who will be the winner. (So, F(1)=1; F(2)=1; F(3)=3; F(4)=1).

(#8-47) This 1-story 5-room house has 16 doors: exactly 1 door ("D") in each of its 16 walls. Is it possible to start at some point, inside or outside the house, and then walk through each door exactly once, and finish at any point (not necessarily where you started; you may visit same room more than once, of course).

```
+------D------+------D------+
|             |             |
|             |             |
D             D             D
|             |             |
|             |             |
+---D---+--D--+--D-+----D---+
|       |          |        |
|       |          |        |
D       D          D        D
|       |          |        |
|       |          |        |
+---D--------D---------D----+
```

(#8-48)) Tigger, Piglet, Pooh, and Eeyore came to a bridge. They have 1 flashlight. It's dark, so nobody can walk without the flashlight. Anyone can walk either alone, or together with another animal, but no more than 2 animals can walk together. It takes Tigger 1 minute to cross the bridge (walking either way); Piglet - 2 minutes; Pooh - 5 minutes; Eeyore - 10 minutes. Any 2 animals together walk at the speed of the slower one. Find the fastest way for them to cross the bridge.

(#8-49) In a triangle ABC, AC is the smallest of the 3 sides. K is a point inside segment AB; L is a point inside segment CB, and $|KA|=|AC|=|CL|$. Let M be the point where segments AL and KC intersect. I is the center of a circle inscribed into the triangle ABC. Prove that lines MI and AC are perpendicular to each other.

(#8-50) Does there exist a set of five natural numbers so any pair is relatively prime and the sum of any subset (of 1 or more) is a composite number?

(#8-51) You are buying a computer and have 3 to choose from: American (**A**), Chinese (**C**), and Russian (**R**). All 3 look alike. **A** always gives the right answer; **C** always gives the wrong answer; and **R** gives unpredictable (random) answers - sometimes right, sometimes wrong. You want to buy either **A** or **C** (you don't care which as long as it's not **R**), and you may address any one computer only, and may only ask it a single question which can be answered with either "Yes" or "No", after which you have to choose your computer. (Computers themselves know which one is which).

(#8-52) Are there any **n** for which there exists a set of **n** points: $A_1, A_2, A_3, \ldots A_n$ on a plane such that when you draw the following segments: A_1A_2, A_2A_3, A_3A_4, ..., $A_{(n-1)}A_n$, A_nA_1, then each of these segments would intersect another one of these segments at exactly 1 point (not counting the segment's ends).

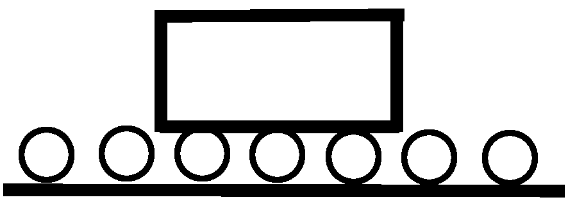
(#8-53) A square board 2005x2005 is filled with numbers. Absolute value of each is not greater than 1. The sum of 4 numbers in any 2x2 square equals 0. Prove that the sum of all numbers is not greater than 2005.

(#8-54) Prove that for any natural number $\boldsymbol{m}$, $\boldsymbol{m}(\boldsymbol{m} + 1)$ is not a power of an integer.

(#8-55) A student plotted the function y= **f** (x) on a piece of paper, with "x" axel being horizontal (increasing left to right), "y" – vertical (increasing bottom to top). He then turned the paper 90° clockwise while centered at the point (0,0), and observed that the plot remained exactly the same. Give an example of such function **f**. Also, prove that equation **f**(x)=x has exactly one solution.

(#8-56) There are 3 boxes: one containing apples, the second one - oranges, the third - a mix of both apples and oranges. They are labeled “apples”, “oranges”, and “mix”, however, none is labeled correctly. You can remove one fruit at a time from any box. What’s the minimum total number you’d need to remove to figure out the contents of each box?

(#8-57) Amtrak decided to re-invent the wheel. They removed all wheels from railroad cars, changed their name to “the circular shweels”, and placed a lot of them all along the rail track. Now, the new rail car is simply a box which glides along on those free-rolling circular shweels, as follows:



This saves money because new trains are cheaper, and they use less fuel due to reduced friction.

MBTA wants to do the same thing, but, unfortunately, Amtrak already got a patent for circular shweels. To avoid the patent, MBTA changed the shape of the shweel to something other than a circle, and called it “the non-circular shweel”. Their cars always stay at a constant height, just like with Amtrak’s circular shweels, so the train rides as smoothly as Amtrak’s (in other words, each non-circular shweel’s width remains the same as it rolls along the track). As an added benefit, since non-circular shweels are not circles, they tend to stay in place until a train comes, instead of wildly rolling around on their own, as do Amtrak’s ones. Give an example of the shape of the non-circular shweel.

(#8-58) Given 3 infinite sequences of natural numbers

$a_1, a_2, \dots, a_n, \dots$

$b_1, b_2, \dots, b_n, \dots$

$c_1, c_2, \dots, c_n, \dots$

Prove that there exist 2 numbers p and q such, that $a_p \geq a_q$; $b_p \geq b_q$; and $c_p \geq c_q$.

(#8-59) A 6x6 board is covered with 18 2x1 dominos, each occupying 2 adjacent squares. Prove that you can cut the board with a straight line into two parts without cutting any dominos.

(#8-60) If $(x+1)^{100}$ is multiplied out, how many of the coefficients are odd?

(#8-61) A team of n students is playing a game: they all stand in a single line (one behind another), and the teacher then hangs a pendant (which is either black or white) on each student's back. Nobody can see his own pendant, and is not allowed to talk, but can see all the pendants in front of him (but none behind him). Each student must guess the color of his own pendant by naming it. They must speak one at a time in any order, until all have spoken. When speaking, a student introduces himself, so they all know which one is speaking; each must only make 1 guess. The team wins $1 for each correct guess. Before the game begins, the students are allowed to get together and work out a game plan. Find the maximum "guaranteed" amount which they can win (which is what they'll win in the "worst-case scenario"; for example, if their plan was to simply make n random guesses, then the guaranteed amount would be 0).

(#8-62) In an isosceles triangle ABC, AB=BC, M is a point in the middle of AC. H is a point on BC, and MH is perpendicular to BC. Point P is the middle of MH. Prove that AH is perpendicular to BP.

(#8-63) How many ways are there of spelling out "ABRACADABRA" by traversing the following diamond, always going from one letter to an adjacent one?

```
          A
        B   B
      R   R   R
    A   A   A   A
  C   C   C   C   C
A   A   A   A   A   A
  D   D   D   D   D
    A   A   A   A
      B   B   B
        R   R
          A
```

(#8-64) Show that there are no positive integers n for which $n^4 + 2n^3 + 2n^2 + 2n + 1$ is a perfect square. Are there any positive integers n for which $n^4 + n^3 + n^2 + n + 1$ is a perfect square? If so, find all such n.

(#8-65) On an island, 2/3 of all men are married, and 3/5 of all women are married. What percentage of island's population is married?

(#8-66) 10 mathematicians are riding on a subway train on the Circular Line in the Moscow Metro (which is in fact a circle, and thus has no last stop) while each wearing either a black hat, or a white one. Everyone can see 9 hats (all hats except his own). At a stop, another mathematician walks in, and puts a poster on the wall which reads: “Each of you 10 people is wearing either a black hat, or a white one. Just to let you know, there is definitely at least one black hat. If at any time any one of you figures out the color of his own hat – and I mean he must know it for sure, and be able to absolutely prove it! – then he must get off this train at the very next stop (except that, for safety reasons, if any person figures out his color during the actual time when the train is standing at a station, he must then wait until the next stop to get off). You guys are not allowed to talk, or make signs, or communicate in any way, all you gonna be doing is just keep sitting here and thinking. Remember, since all of you are, like, “totally smart”, you will all be thinking pretty much alike. Whoever exits the train according to these rules, gets $1,000.”

Now, assuming there were actually 7 black hats, describe what’s going to happen.

(#8-67) Find a positive integer m such that $m/2$ is a perfect square and $m/3$ is a perfect cube. Can you find a positive integer n for which $n/2$ is a perfect square, $n/3$ is a perfect cube and $n/5$ is a perfect fifth power?

(#8-68) A quadrilateral is called "concave" if it has an angle greater than 180 deg. Are there any numbers N>2 such that there exists a convex N-polygon which can be cut into several quadrilaterals, all of which are concave?

(#8-69) Numbers 1, 2, 3,..., 2n are divided into two groups, n numbers in each group. Let $a_1, a_2, \dots, a_n$ be the first group of numbers, arranged in ascending order, and $b_1, b_2, \dots, b_n$ be the second group of numbers, arranged in descending order. Prove that $|a_1-b_1|+|a_2-b_2|+\dots+|a_n-b_n| = n^2$.

(#8-70) In the state of Shmassachusetts, the total income earned by 10% of those people who have the highest income equals 90% of the total income of everyone in the state. Shmassachusetts is divided into several counties. If you select any 10% of the population of any given county, then their income will not exceed 11% of the total income of everyone in that county. Is this possible?

(#8-71) We have a simple balancing scale with 2 trays (and no extra weights), and a set of 15 coins, which all look identical. We know that there is exactly one fake coin among them, and that it's either a little bit lighter, or a little bit heavier than a genuine coin. We also know that coin #1 is genuine. Can you find the fake coin by using the scales no more then 3 times?

(#8-72) $\mathbf{p}$ is a prime number, $\mathbf{a}$ is any number, $1 \le \mathbf{a} \le (\mathbf{p}-1)$; prove that $\mathbf{a}^{(\mathbf{p}-1)}-1$ is divisible by $\mathbf{p}$.

(#8-73) Is it possible to cut a square into several triangles in such a way that all of these triangles' angles would be acute?

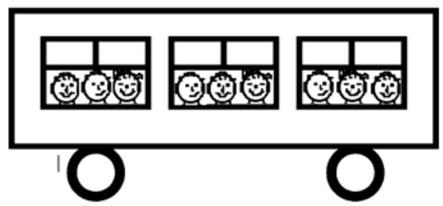
(#8-74) This is a 2-D problem, so assume that "altitude" = 0. A spy airplane circles point A at the distance 10 miles and at the constant speed of 1000 miles/hour. A missile is fired towards the airplane from A flying at the same speed as the airplane, and it flies in such a way that it is always on the straight line between A and the airplane. How much time will pass before the missile hits the airplane?

(#8-75) There are 3 piles consisting of 5, 49, and 51 stones. At any point, you may choose any two piles and combine them to make a single pile. Or, instead, if there exists a pile with an even number of stones in it, then, if you prefer, you may divide it into two equal piles. Is there a way to end up with 105 piles, each consisting of 1 stone?

(#8-76) **p** is a prime number; **r=(p-1)!** [that is, **r**=1•2•…•(p-1)]. Prove that of the two numbers: **(r+1)** and **(r-1),** one is divisible by **p**.

(#8-77) Below is a photo of a school bus in Boston, Massachusetts (the school is to the left, and children's homes are to the right). Can you tell what time it is, knowing that it is Monday, either 8 AM, or 3 PM ?

SCHOOL «-------------------



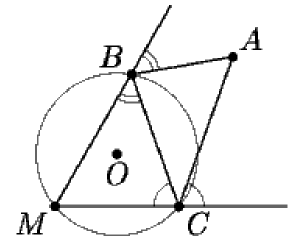
---------------------» HOMES

(#8-78) Is it possible to place 4 players on a football field so that the 6 distances between them would equal 1, 2, 3, 4, 5, and 6 yards?

(#8-79) In a group of 17 people, when 2 of them meet, they talk about 1 of 3 subjects (and this subject remains the same forever for each given pair or people): weather, sports, or politics. Prove that you can find 3 of them who, if they meet, can talk all together (that is, all 3 pairs will talk about the same subject).

(#8-80). A group of soldiers is assigned to guard a tent (which is a 1x1 yard square) during a dark night. Each soldier has a headlamp which illuminates a straight segment 100 yards long right in front of him. Once in position, the soldiers are not allowed to move, nor to turn their headlamps. Is there a way to place some finite number of soldiers so that the enemy won't be able to sneak in on either the tent, or on any soldier.

(#8-81) There is a billiard ball at point A inside an acute angle with a vertice M. We shoot the ball from A so that it bounces off one side of the angle at point B, then bounces off its other side at point C, then returns to point A. O is the center of the circle circumscribed around the triangle MBC. Prove that point O lies on the straight line connecting points M and A.



(#8-82) On a 100x99 chessboard, draw its main diagonal. W is the sum of the lengths of those parts of this diagonal which go through white squares; B is the same thing for black squares. Find the value of $\frac{W}{B}$. Also: same question for the 101x99 board.

(#8-83) Pooh is going to walk from his home to the Piglet's house for a sleepover. He left home Friday at 6 pm and arrived at the Piglet's house at 8 pm. On Saturday, he left the Piglet's house at 6 pm, walked along the same path, and arrived home at 8 pm. Prove that there exists a point on his route such that he was at that point on Saturday at exactly the same time as on Friday.

(#8-84) 24 couples meet at a party, and start hugging each other (except that nobody ever hugs his/her own partner). One of the 24 couples are the Host and the Hostess. At some point the Host tells everyone to stop hugging, and asks each of the 47 people how many people they have hugged so far. Everyone gives a different answer. How many people has the Hostess hugged?

(#8-85) Pooh and Piglet are lost in a forest. Pooh is carrying 2 bags, one with black beans, the other - with white beans. They agreed in advance that if they were to become separated, then Pooh would walk in a straight line, and drop beans behind him. When Piglet finds Pooh's trail of beans, he wants to also be able to tell which way Pooh went (right or left), so he could then try and catch up with him. What is the shortest repeating binary pattern (white bean means "0", black one means "1") they can agree on which Pooh can use in order to communicate to Piglet in which direction Pooh was walking?

(#8-86) There are 2004 points on the plane. Prove that you can cover them with a set of circles, whose diameters, when added together, total less than 2004, and the distance between any two of which is more than 1. [The distance between 2 circles which have radii **r** and **s**, and their centers being a distance **d** apart is the greater of 0 and **d - r - s**].

(#8-87) In a park, the distance between any two trees is less than the difference between their heights. The tallest tree is 1002 ft tall. Prove that you can build a fence around this park (to keep bears out so they won't damage trees), the length of which is less than 2005 ft. Note that the fence can be any shape, as long as all trees are on the inside of the fence, so bears can't get to any tree.

(#8-88) Let **A** be some set of ten different two-digit natural numbers. Prove that one can always find 2 subsets of **A**, each consisting of at least 3 of these numbers, so that if one adds up all numbers within each of these subsets, these 2 sums will be the same.

(#8-89) Given a balancing scale with 2 trays and no extra weights, and 3 very large identical bottles. One of the bottles has 1 oz of water in it. How can you fill one bottle with exactly 85 oz of water from a garden hose if you are allowed to use the scale no more than 8 times?

(#8-90) An infinite number of children are standing in a single line at the cafeteria. Prove that the Principal can order some children (which may be either a finite, or an infinite number) to leave so that there will still be an infinite number of children remaining, and that they will all be arranged in order of their height (either ascending, or descending).

(#8-91) There are two points A and B on the plane. All you have is one compass (no ruler, nor anything else; and the segment AB is not drawn). Can you find point M which is the middle of the segment AB ?

(#8-92) A rectangular table m x n is filled with numbers 1 and -1. There are at least 2 "1"s, and at least 2 "-1"s. Prove that there exist 2 columns and 2 rows such that the sum of 4 numbers at intersections of these rows and columns equals 0.

(#8-93) Given 601 sets; each consists of exactly 25 elements; the union of any 2 of them consists of exactly 49 elements. What could be the number of elements in the union of all 601 sets? (Find all possible answers.)

(#8-94) Prove that for any prime $p \geq 7$ the number $p^4 - 1$ is divisible by 240.

(#8-95) In a country there are 2004 towns, some are connected by direct airline flights so that you can reach any town from any other (perhaps, with some connections). Each direct flight takes less than 1 hour. Given any 2 towns, the total flying time on the shortest route between them is also less than 1 hour. After one of the direct flights was canceled, it turned out that it is still possible to reach any town from any other. Prove that now, for any 2 towns, the total flying time on the new shortest route between them is less than 3 hours.

(#8-96) Prove that the following is never true for any set of 4 natural numbers a, b, c, and d: $\mathbf{3a^4 + 5b^4 + 7c^4 = 11d^4}$

(#8-97) The circus ring (a circle) is illuminated by n lights. Each light illuminates a convex figure. If any 1 light is turned off, then the entire ring will still be illuminated. But if any 2 lights are turned off, then some part of the ring will no longer be illuminated. Find all n for which this is possible.

(#8-98) On an infinite sheet of graph paper where the size of each square is 1x1, an infinite set of 2x1 dominos is placed so that each domino covers 2 cells, and every 1x1 cell is covered. Can this be done in such a way that for every graph line (those are all the straight lines which make the graph paper) there would exist only a finite set of dominos which are cut in half by that line?

(#8-99) Given a circle and a square, is it possible to paint all points of the circle, and all points of the square, using 2 colors (white and black), so that the set of white points of the circle would be similar to the set of white points of the square, and the set of black points of the circle would be similar to the set of black points of the square (the 2 similarity coefficients don't have to be equal).

(#8-100) There is $500 in Pooh's bank account. He has no other money, but he has an ATM card. The ATM machine offers a choice of only 2 transactions: either withdraw $300, or deposit $198, and it won't allow you to overdraw the account, but there is no limit on the number of transactions. What is the maximum amount Pooh can withdraw?

(#8-101) Each natural number is assigned 1 of 2 colors: blue or red. Prove that there exists a color, such that for every natural k there exist an infinite number of natural numbers of that color which are divisible by k .

(#8-102) In a deck of 52 cards, some cards are positioned face-up, and others are face-down. From time to time, the player selects such contiguous part of this deck (a set of 1 or more adjacent cards) in which both the top and the bottom cards are currently positioned face-up (or, if there is only 1 card in the set, then it must be face-up). He then slides this set out, turns the entire set upside-down as a whole (without flipping individual cards), and then re-inserts it back into the exact spot in the deck where he removed it from. Prove that no matter how he plays, eventually (after a finite number of moves) the game will end (meaning: all cards in the deck will be positioned face-down, so it won't be possible to make another move).

(#8-103) 2 different numbers x and y (not necessarily integers) are such that $x^2 - 2004x = y^2 - 2004y$. Find the number $x+y$.

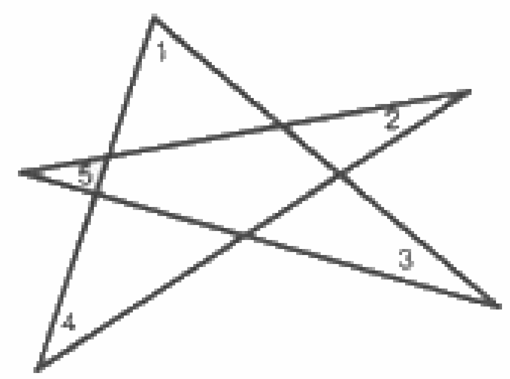
(#8-104) Is it possible to put a sequence of 100 points on the plane (first put the first one, then the second one, and so on) so that no 3 of them belong to the same straight line, and so that at any moment the current set of points would have a line of symmetry?

(#8-105) In a game for 2 players, in which they alternate making moves, each move consists of adding 1 pawn to a 31x31 board (which is empty in the beginning). There must never be more than 2 pawns in any column, nor in any row. The player who cannot make his next move loses. Which player has a winning strategy?

(#8-106) There is a set of n coins, each weighing a rational number of grams. If any 1 coin is removed, then the remaining n-1 coins can be divided into 2 groups of equal weight. Prove that all n coins have the same weight.

(#8-107) In the 3-D space, every point is assigned 1 of 4 colors. Prove that there exist 2 points of same color such that the distance between them equals 1 foot.

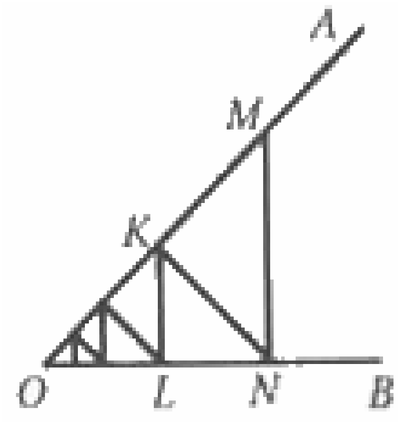
(#8-108) Find the sum of the 5 internal angles of a 5-angled star:



(#8-109) Give an example of a polynomial P(x) having the degree of 2005 such that for any x P(x)+P(1-x)=1.

(#8-110) Find the smallest natural numbers a and b such that b>1 and $\sqrt{a\sqrt{a\sqrt{a}}} = b$

(#8-111) Point M is on the side AO of the angle AOB. From point M we draw a segment MN, perpendicular to line OB (point N is on OB) Then from N we draw

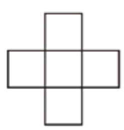


a segment NK, perpendicular to AO, then segment KL perpendicular to OB, etc.

Using a compass and a ruler, draw a segment, the length of which equals the infinite sum of the lengths of segments MN, NK, KL, etc.

(#8-112) A team of 22 football players received a set of 22 shirts, individualized with their names. After putting them on, they discovered that everyone had a wrong shirt. The players then sat down on the field, in a single row. You can tell any 2 neighbors to swap their shirts, but only if neither of them is wearing his own shirt at that moment. Will you always be able to help everyone get his own shirt with a few such swaps?

(#8-113) Tigger, Piglet, Pooh, and Eeyore are at Pooh's house. They are going to Kanga's house which is 33 miles away. They have a 2-seat scooter which rides at 25 miles per hour with 1 animal, or at 20 miles per hour with 2 animals. Each animal walks at 5 miles per hour. Prove that they can make it in less than 3 hours.

(#8-114) On an infinite sheet of graph paper each cell is assigned 1 of 5 colors. In every figure of this type  all 5 colors are different. Prove that in any figure of this type  all 5 colors are also different.

(#8-115) **A** Pooh came to a party at Piglet's house, and brought a barrel with 18 gallons of wine in it. Also, Eeyore came and brought two empty 7-gallon buckets and one empty 4-gallon pot. Will they be able to divide the wine into 3 equal portions (so as to end up with 6 gallons in the barrel, and 6 gallons in each of the two buckets)?

B When they finished drinking all the wine, Pooh went home and then brought another barrel, again with 18 gallons of wine, except this one had a narrow top, so the buckets didn't fit inside it. It was also heavy, and impossible to tilt, so the only way to get the wine out of the barrel now was with the 4-gallon pot. Can they again divide the wine into 3 equal portions?

(#8-116) Every point in the 3-D space is assigned 1 of 5 colors (so there is at least 1 point of each of the 5 colors). Prove that there exists a plain which contains points of at least 4 colors.

(#8-117) In a sequence of 2004 numbers $\{a_1, a_2, \ldots, a_{2004}\}$ half of the numbers are written with a blue pen, the rest – with a red one. The sequence of blue numbers (which is what would be left if one erased all red numbers) is all numbers from 1 to 1002 written in ascending order. The sequence of red numbers is also all numbers from 1 to 1002, but written in descending order. Prove that the subsequence $\{a_1, a_2, \ldots, a_{1002}\}$ contains all numbers from 1 to 1002.

(#8-118)

(#8-119)

(#8-120) Segment AB is the diameter of circle #1. Point A is the center of circle #2 which intersects the segment AB at point C, and $|AC| < \frac{1}{2}|AB|$. Line $\mathcal{L}$ touches both circle #1 (at point D), and circle #2 (at point E). Prove that CD is perpendicular to AB.

(#8-121) There is 1 light-emitting point L in an empty 3-D space. Is it possible to place 4 solid spheres so that none of them would touch L, and so that they would completely block all the light coming from L in all directions (the spheres don't have to be of equal size)?

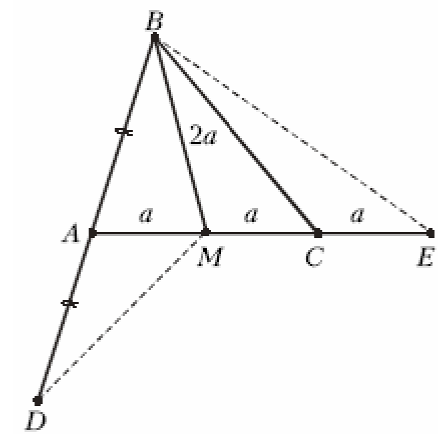
(#8-122) Starting with 2 decks of playing cards, each containing 36 cards, each of which has been shuffled, we place one of them on top of the other, and then find the number of cards between each pair of identical cards in these 2 decks (for example, we count how many cards are between the ace of spades of the top deck and the ace of spades of the bottom deck, and so on). We then add up all of these numbers. Find the result.

(#8-123)

(#8-124)

(#8-125) On a sheet of graph paper there is a convex polygon. All of its vertices are in the points of intersections of grid lines, but none of its sides are parallel to the grid lines. Prove that the sum of lengths of vertical segments of grid lines which lie inside the polygon is equal to the sum of lengths of horizontal segments which lie inside the polygon.

(#8-126) In the triangle ABC, M is the middle of side AC, and |BM|=|AC|. Point D is on the line BA; point E is on the line AC; |BA|=|AD| and |MC|=|CE|. Prove that DM is perpendicular to BE.



(#8-127) There is a regular **2n**-polygon, half of its vertices are blue, and half are red. Teacher starts at some blue vertex and writes numbers **1, 2, …, n** , in order, next to blue vertices going clockwise. Then she starts at some red point, and writes numbers **1, 2, …, n** , also in order, next to red vertices, but this time going counter-clockwise. Prove that you can cut this **2n**-polygon into 2 equal halves by a straight line going through its center and mid-points of 2 opposite sides so that each half will contain all **n** numbers **1, 2, …, n**.

(#8-128)

(#8-129)

(#8-130) reserved for an olympiad as of 5-6